Generative models

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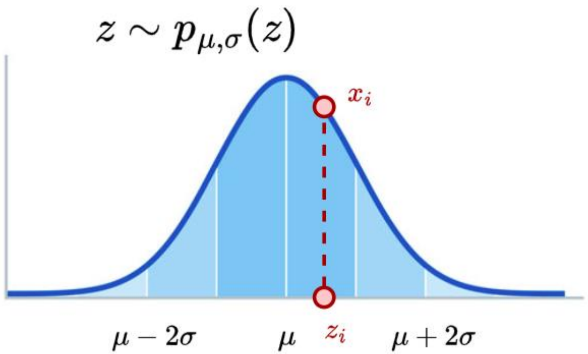
What is the interest of generative models ?

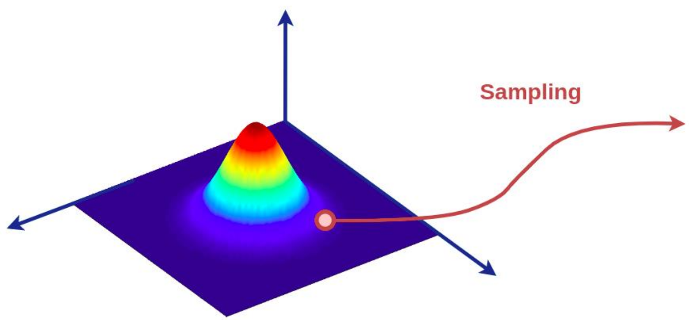
- **How to generate synthetic faces ?**

By modeling the corresponding distribution $p_\theta(\cdot)$!

→ **Reminder: normal distribution**



- **How to model complex distributions ?**



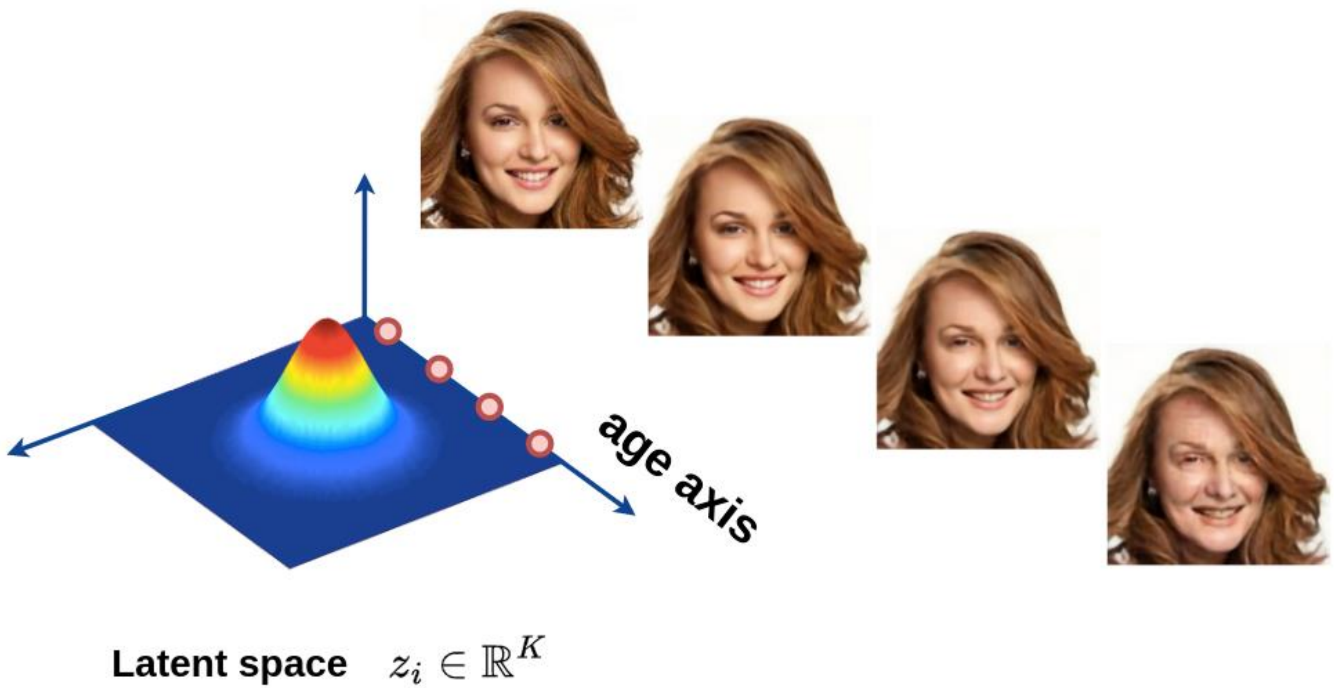
Latent space $z_i \in \mathbb{R}^K$

Face distribution

What are the interest of generative models ?

- **What for ?**

One obsession is to master the latent space !!!



What are the interest of generative models ?

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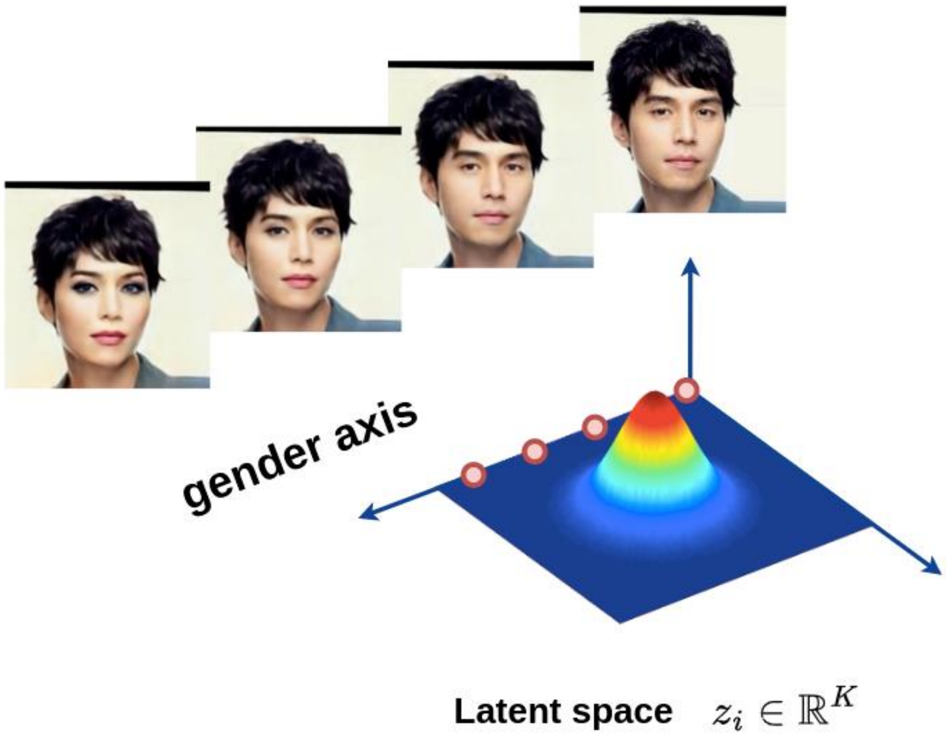
One obsession is to master the latent space !!!

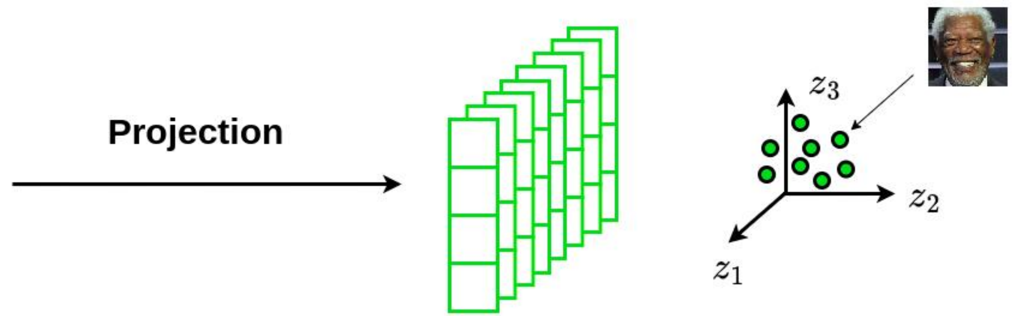


Auto-encoders

How to learn a distribution ?

- **Projection into a simpler, lower-dimensional representation space**

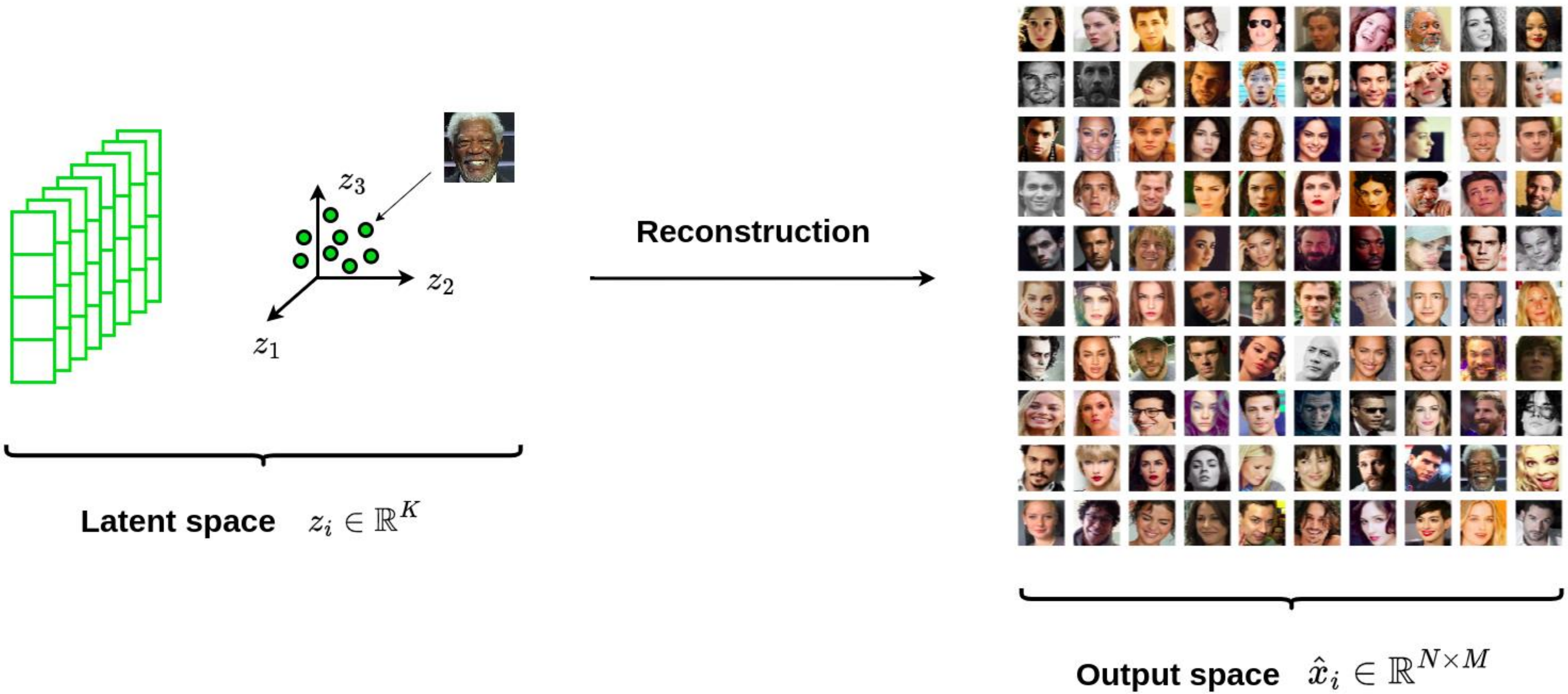
Input space $x_i \in \mathbb{R}^{N \times M}$



Latent space $z_i \in \mathbb{R}^K$

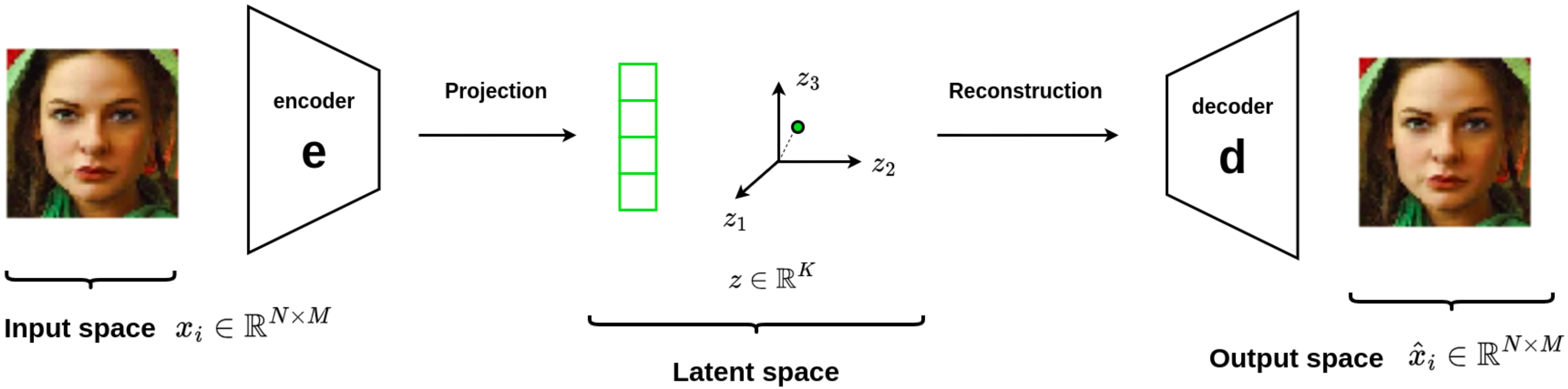
How to learn a complex distribution ?

- **How to have a relevant representation space ?**



Auto-encoder framework

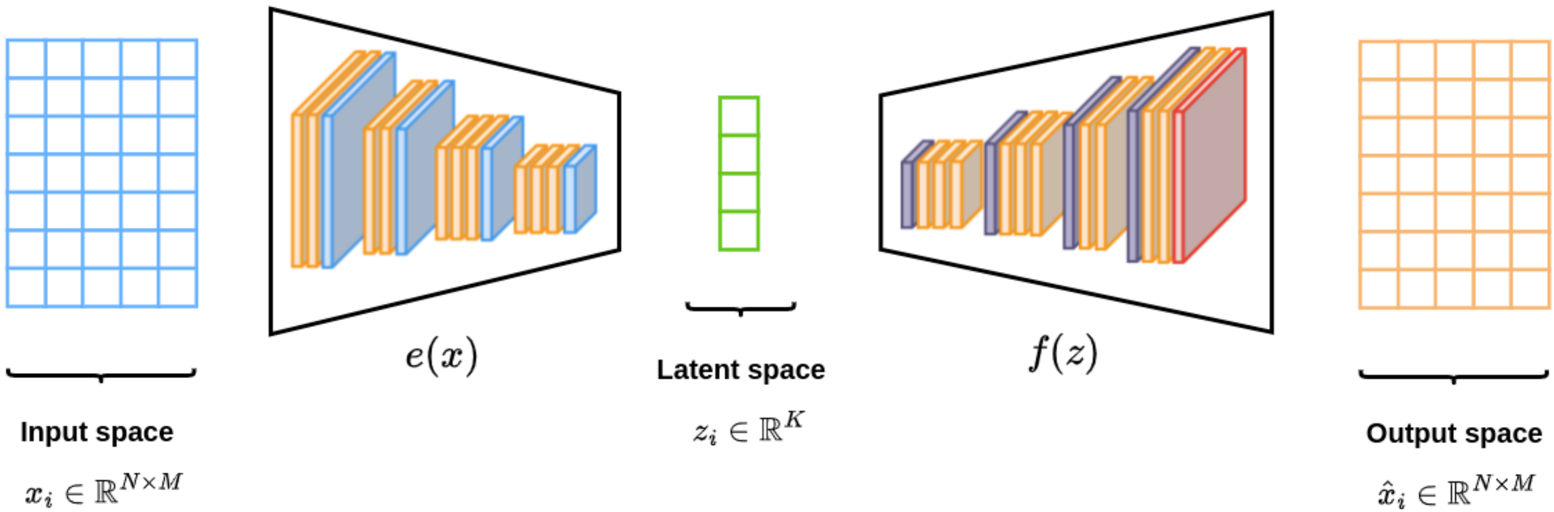
▶ Standard architecture



▶ Deep learning loss function

$$\text{loss} = \|x - \hat{x}\|^2$$

- **Encoder / Decoder modeled through (convolutional) neural networks**

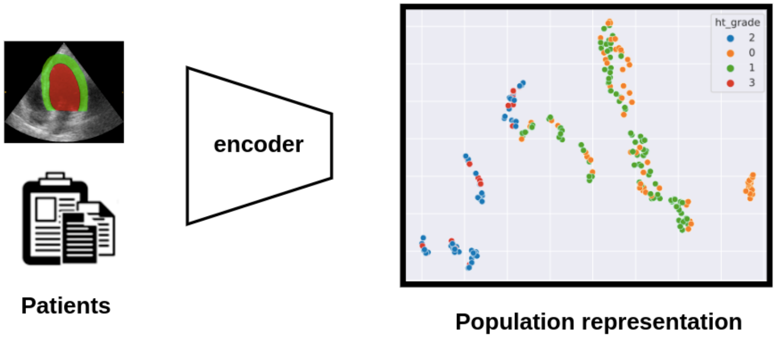


$$\text{loss} = \|x - f(e(x))\|^2$$

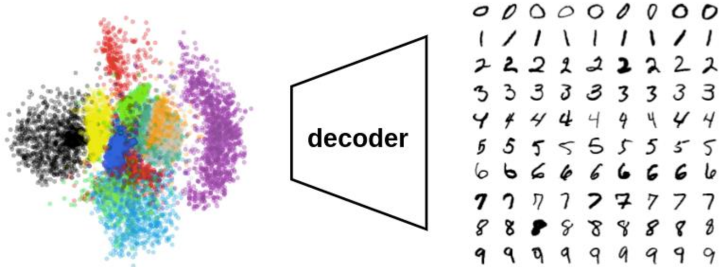
Interest of auto-encoders

▶ Auto-encoder ? For what purpose ?

➔ Data representation

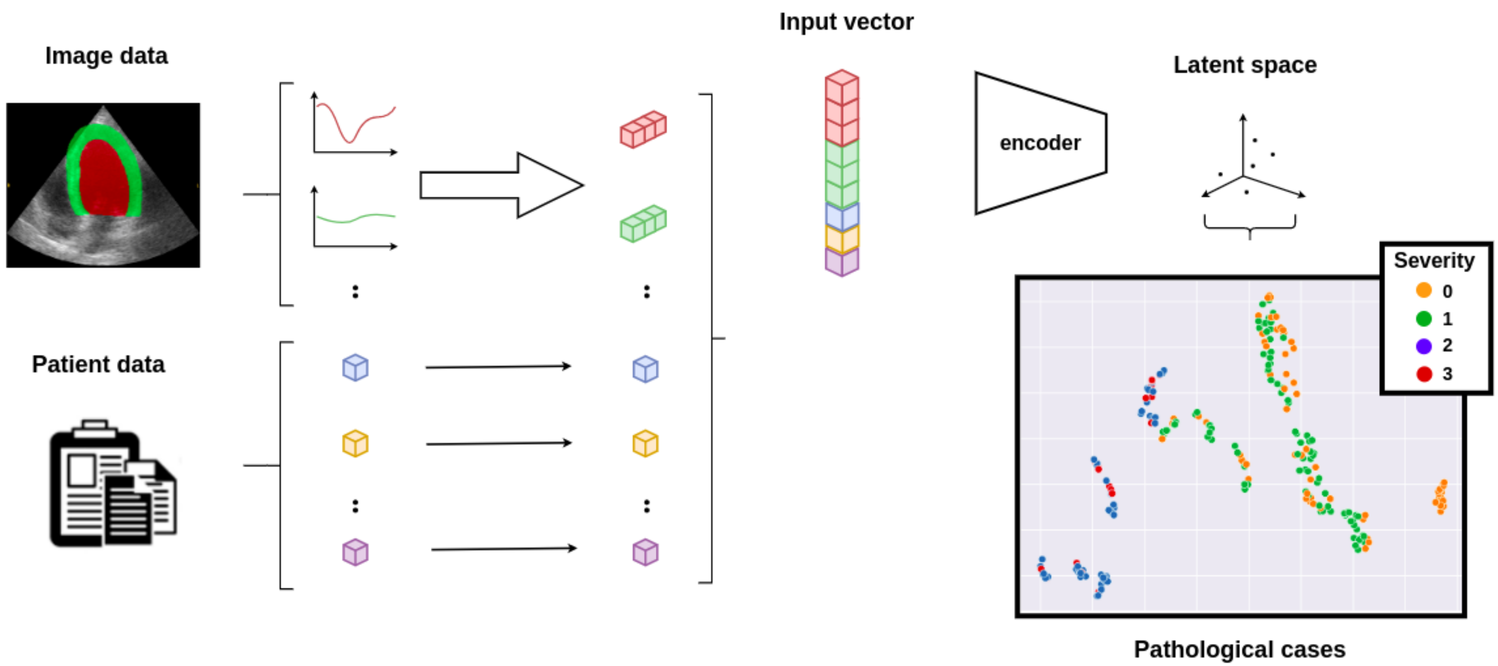


➔ Generative model

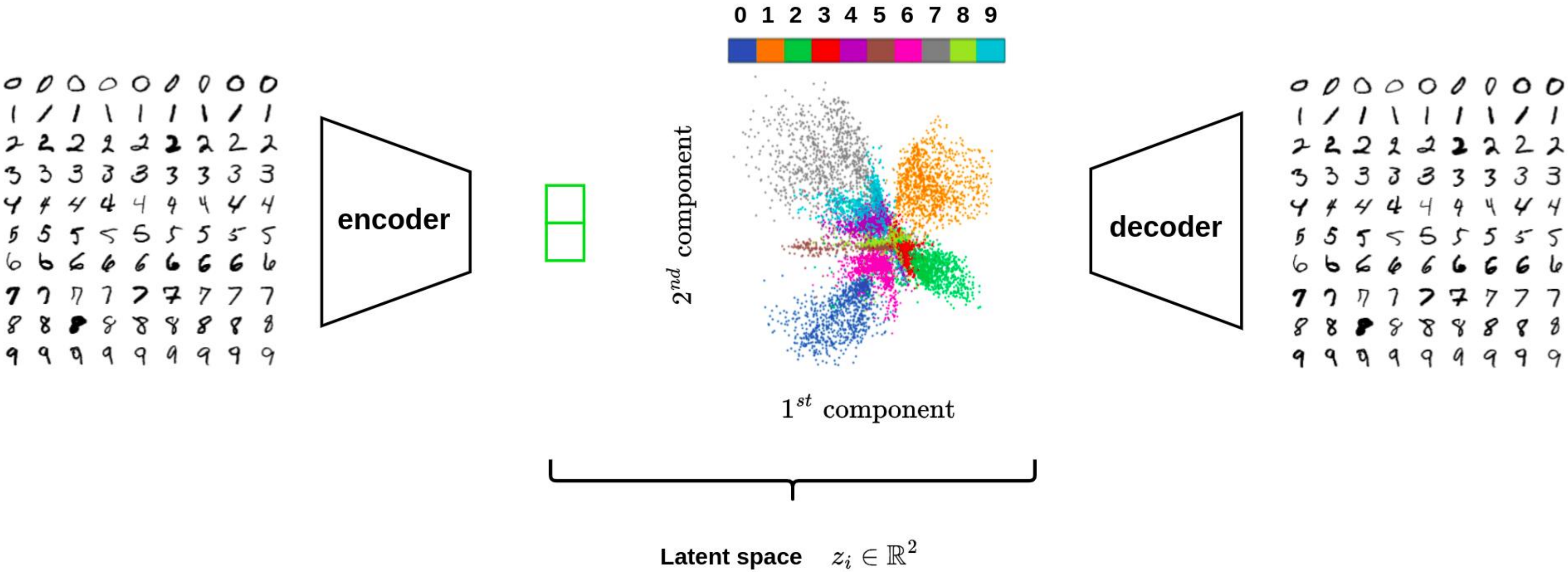


Interest of auto-encoders

- **Data representation**

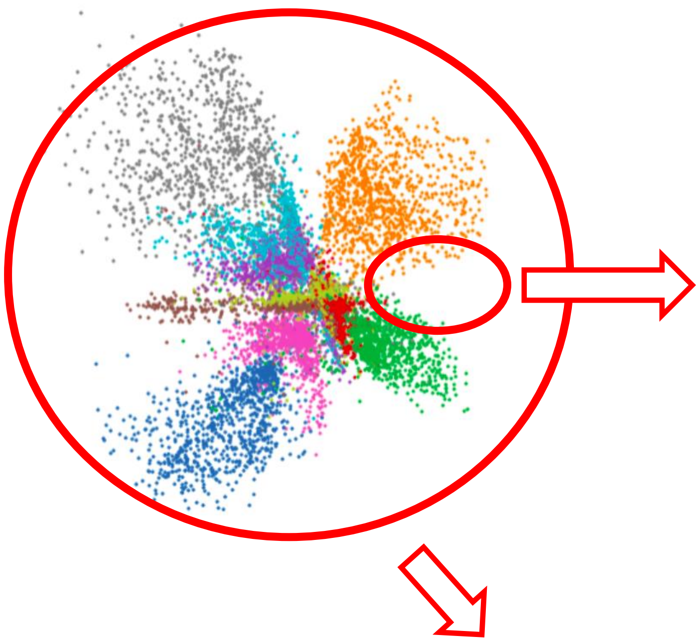


- **Generative model**



Limitations

- **Needs to better control the structure of the latent space**

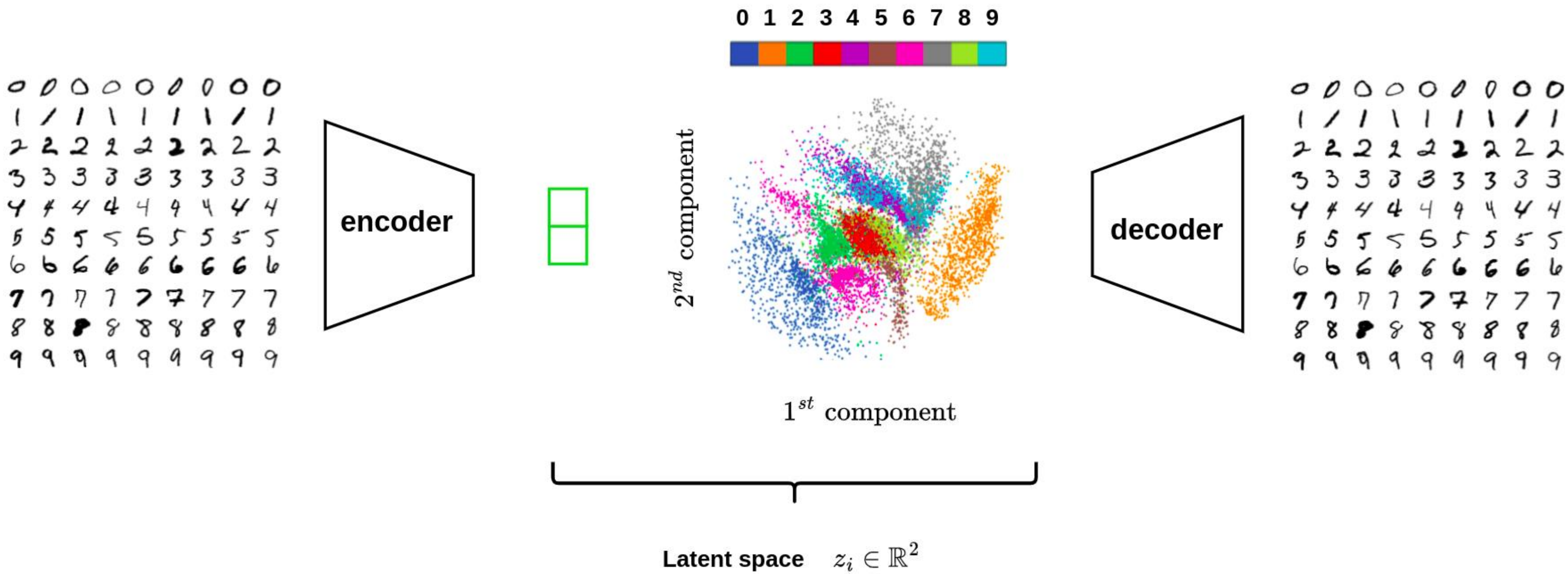


Lack of local continuity

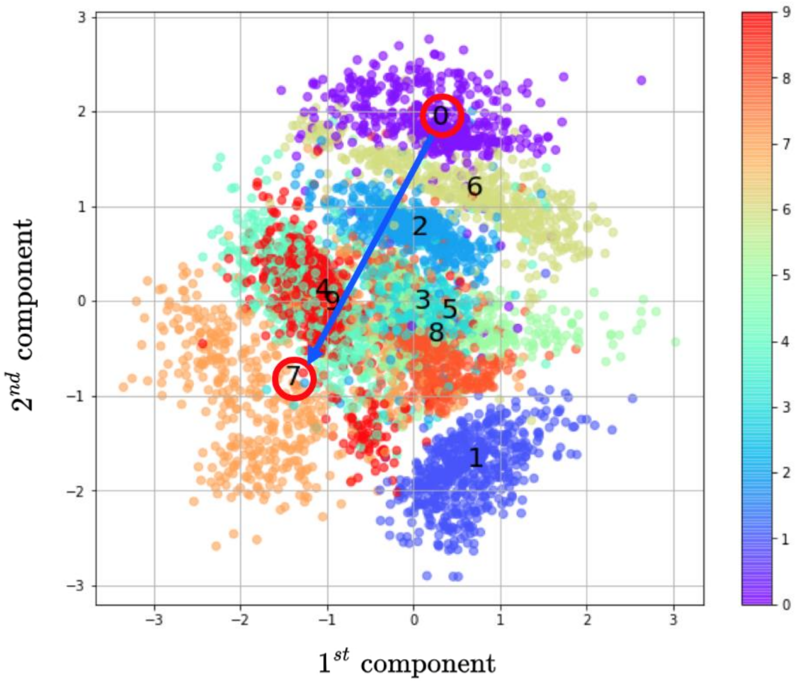
Lack of global completeness

Interest of auto-encoders

- **Generative model with better properties thanks to *variational framework***

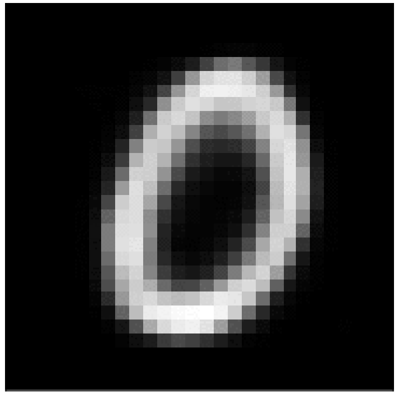


▶ Generative model with variational framework



Linear interpolation into the latent space

$$t \cdot z_0 + (1-t) \cdot z_7, \qquad 0 \leq t \leq 1$$



Variational autoencoders

All the mathematical details are given there !

https://creatis-myriad.github.io/tutorials/2022-09-12-tutorial-vae.html

Key concepts

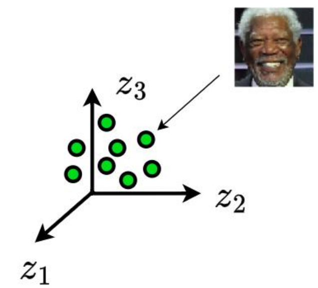
- **Enforcing a structured latent space**
  - → **Through a probabilistic framework**
  - → **By imposing continuity**
  - → **By imposing completeness**

Mathematical formulation

Observation variables

$$\begin{tikzcd} X \arrow[r, bend right, "p(z|x)"'] & Z \arrow[l, bend right, "p(x|z)"'] \end{tikzcd}$$

Hidden variables



Approximation of $p(z|x)$ through a variational inference technique

Probabilistic framework

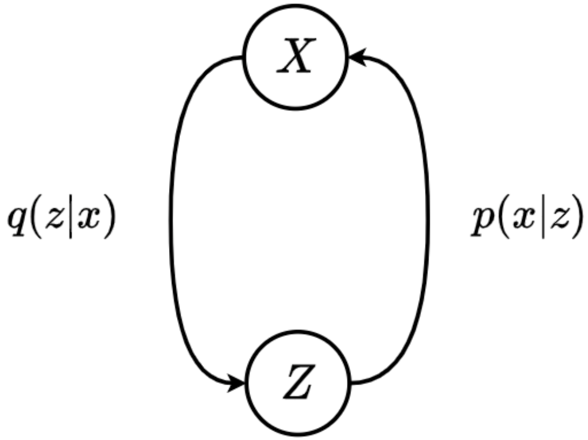
- **Hypotheses**

  - $q(z|x)$ **is modeled by an axis-aligned Gaussian distribution**

  - $q(z|x) = \mathcal{N}\left(\mu_x, \sigma_x\right) = \mathcal{N}\left(g(x), diag(h(x))\right)$

$$\boxed{(g^*, h^*) = \underset{(g,h)}{\arg\min}\; D_{KL}\left(q(z|x) \parallel p(z|x)\right)}$$

$D_{KL}\left(\cdot \parallel \cdot\right)$ *Kullback-Liebler divergence function*

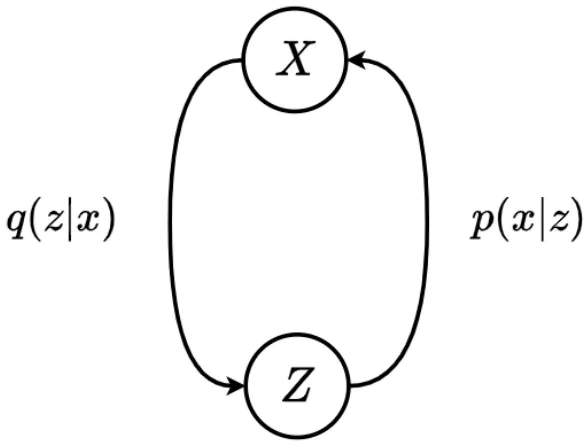


Probabilistic framework

- **Optimization process**

  - **Maximization of the Evidence Lower Bound (ELBO)**

$$\mathcal{L} = \mathbb{E}_{z \sim q_x}\left[log\left(p(x|z)\right)\right] - D_{KL}\left(q(z|x) \parallel p(z)\right)$$



  - **By exploiting gaussian assumption**

$$p(x|z) = \mathcal{N}\left(f(z), cI\right)$$

$$\mathcal{L} \propto \mathbb{E}_{z \sim q_x}\left[-\alpha \|x - f(z)\|^2\right] - D_{KL}\left(q(z|x) \parallel p(z)\right)$$

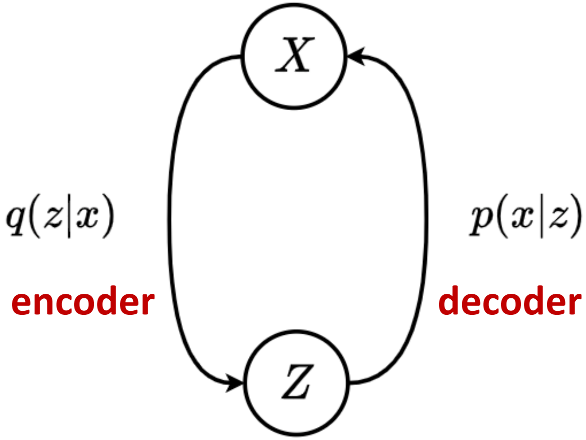
Probabilistic framework

▶ **Optimization process**

$$(f^*, g^*, h^*) = \underset{(f,g,h)}{\arg\min} \left(\mathbb{E}_{z\sim q_x}\left[\alpha \|x - f(z)\|^2\right] + D_{KL}\left(q(z|x) \parallel p(z)\right)\right)$$

▶ **Deep learning loss function**

$$\text{loss} = \alpha\|x - f(z)\|^2 + D_{KL}\left(\mathcal{N}\left(g(x), diag\left(h(x)\right)\right), \mathcal{N}\left(0, I\right)\right)$$



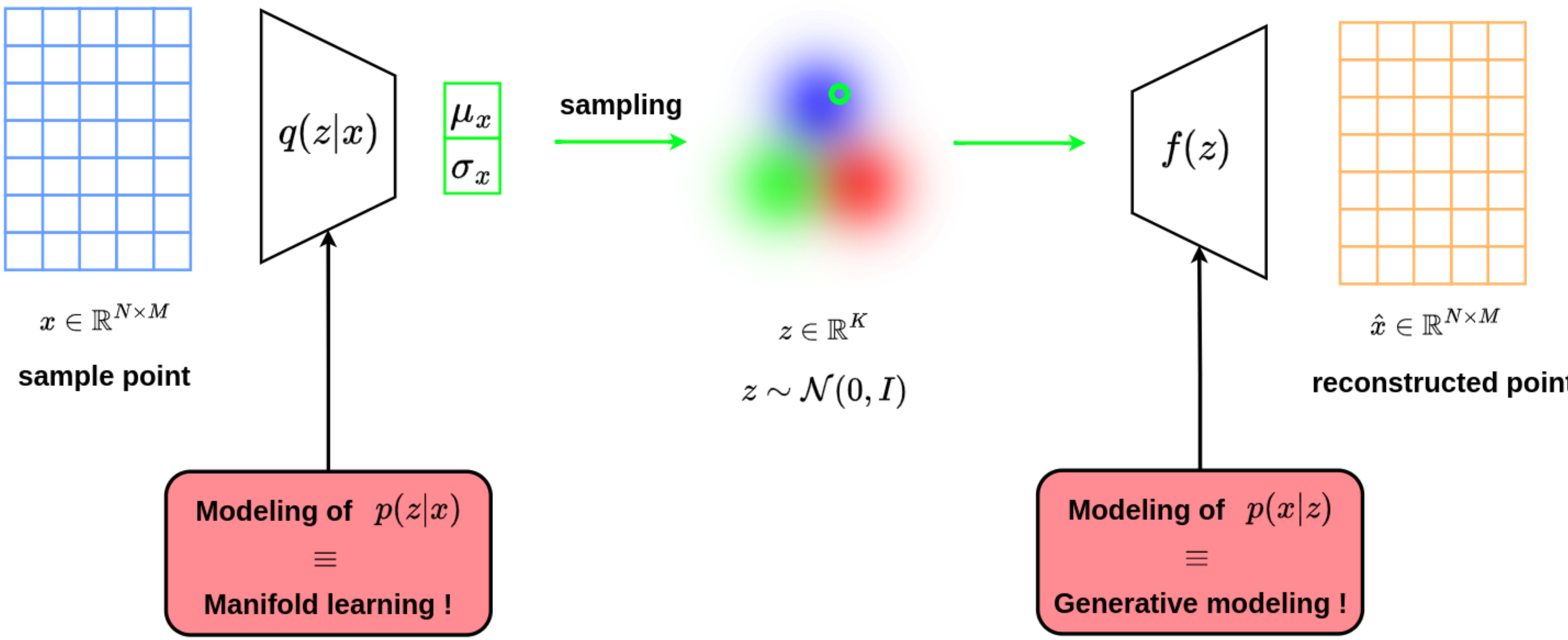
➔ **$g(\cdot)$ and $h(\cdot)$ are modeled through an encoder**

➔ **$f(\cdot)$ is modeled through a decoder**

Probabilistic framework

- **Loss interpretation**

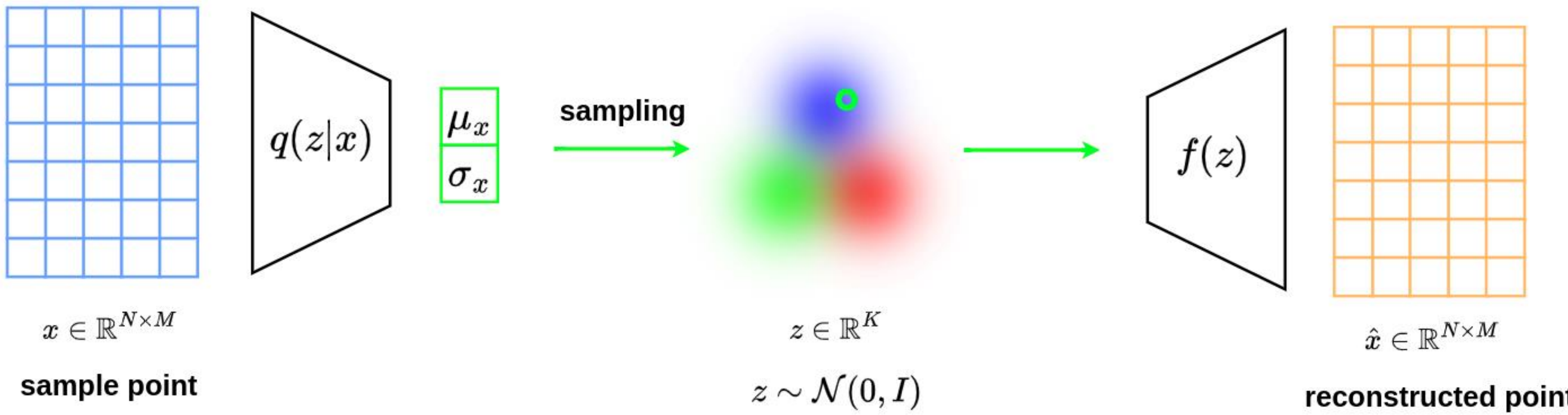
$$\text{loss} = D_{KL}\left(\mathcal{N}\left(g(x), diag\left(h(x)\right)\right), \mathcal{N}\left(0, I\right)\right) + \alpha \|x - f(z)\|^2$$



Probabilistic framework

- **Loss interpretation**

$$\boxed{\text{loss} = D_{KL}\left(\mathcal{N}\left(g(x), diag\left(h(x)\right)\right), \mathcal{N}\left(0, I\right)\right) + \alpha \|x - f(z)\|^2}$$

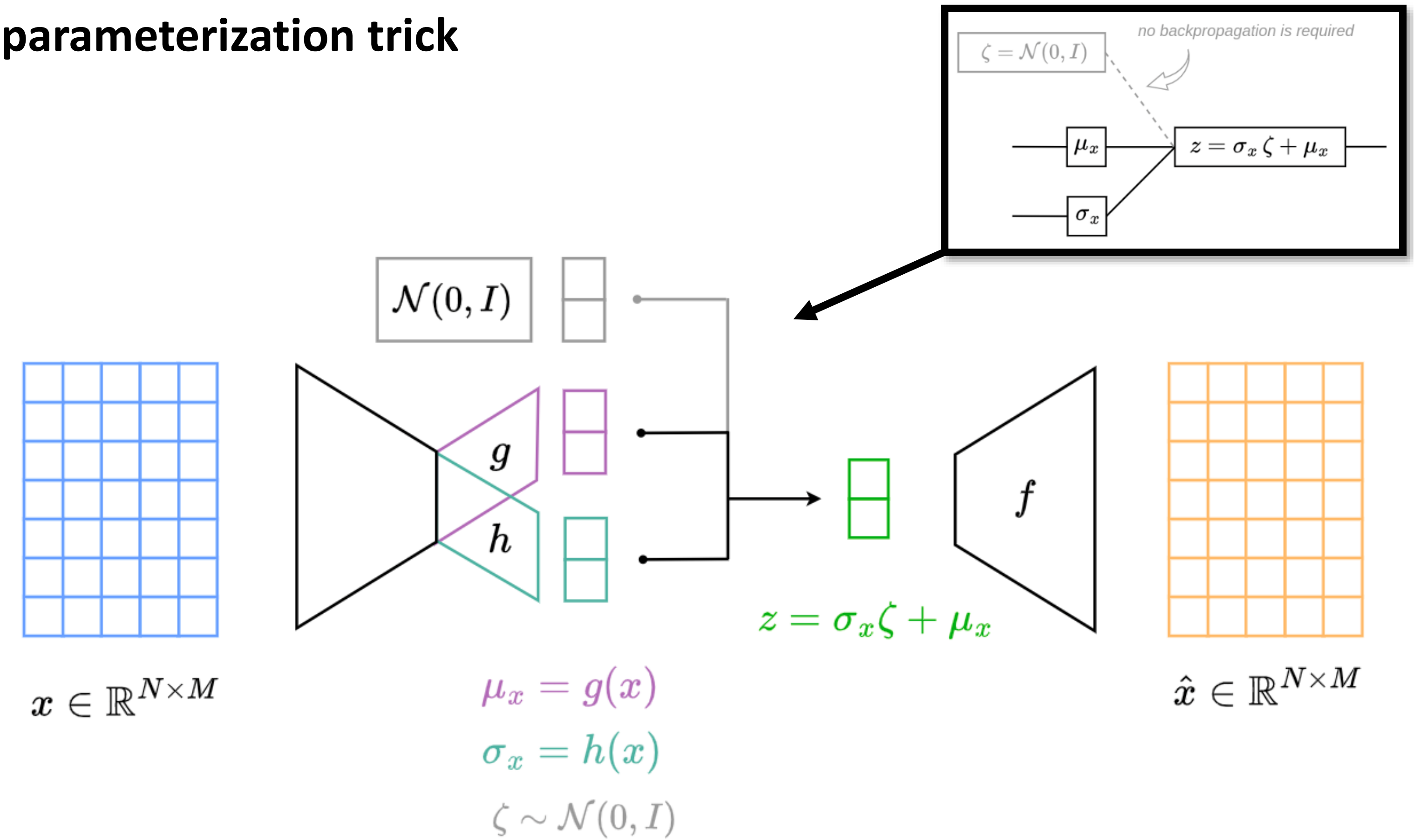


→ $\mathcal{N}\left(g(x), h(x)\right)$ **imposes local *continuity***

→ $\mathcal{N}\left(\cdot, \mathcal{N}(0, I)\right)$ **imposes global *completeness***

Deep learning implementation

▶ Reparameterization trick

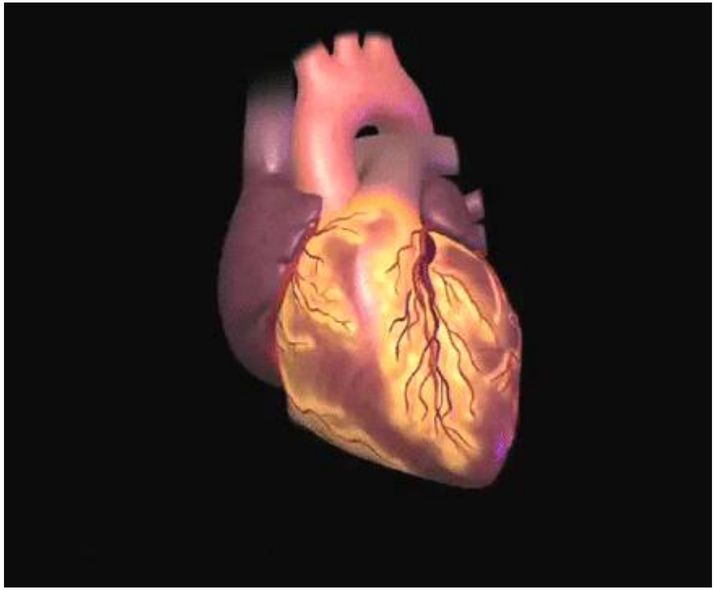


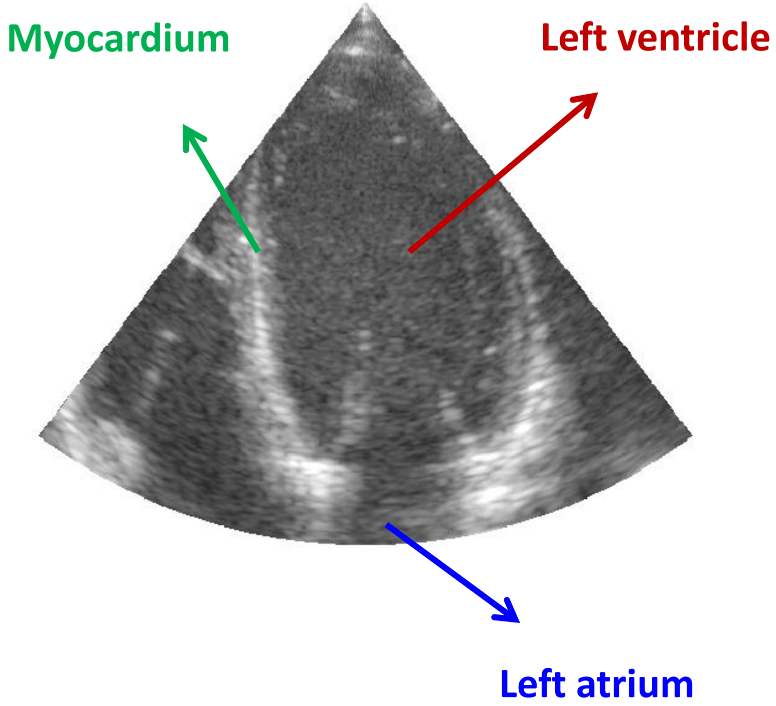
Practical applications

The obsession is to master the latent space !!!

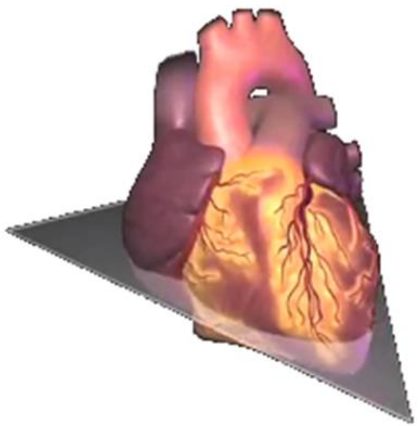
Needs for accurate and robust segmentation of cardiac structures

- **Quantification of clinical indices from echocardiographic images**

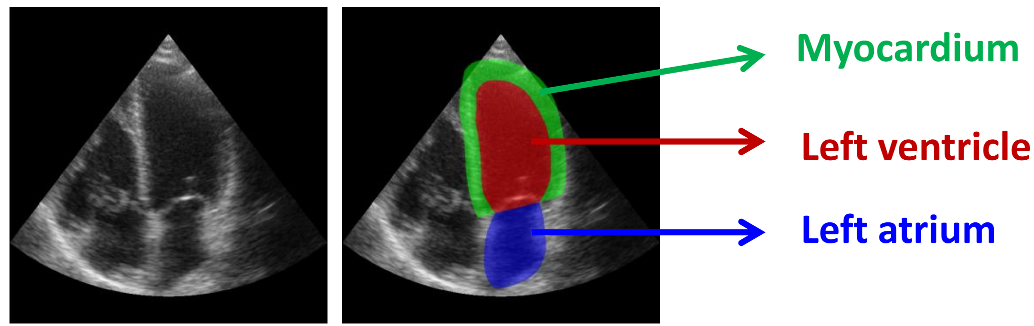




Anatomical clinical indices



Apical views

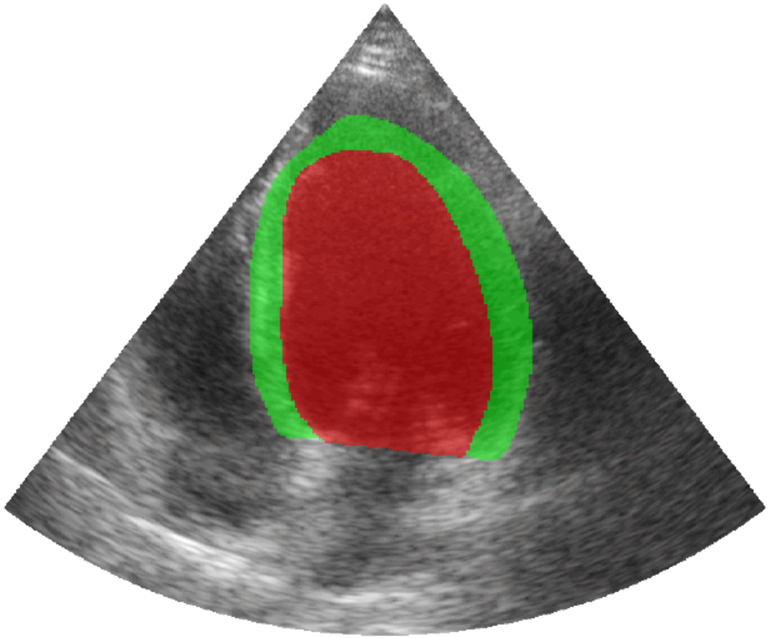


- → Left ventricle volumes
- → Left ventricle ejection fraction

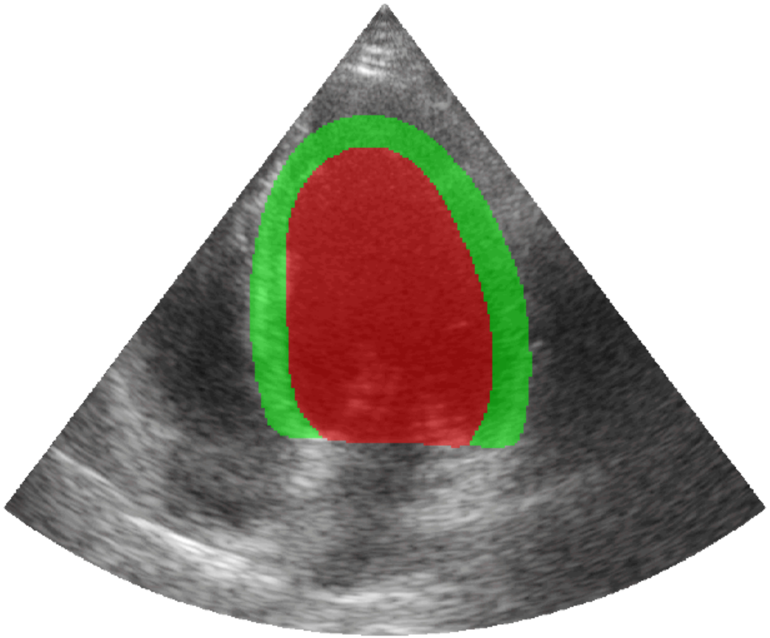
How to guarantee temporal consistency ?

- **Quantification of clinical indices from echocardiographic images**

What we have with a 2D U-Net

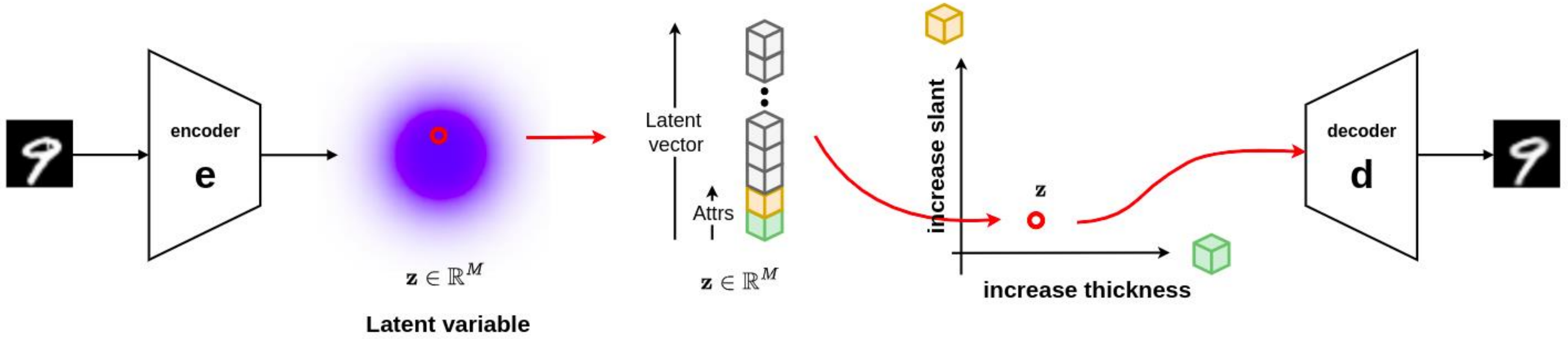


What we want



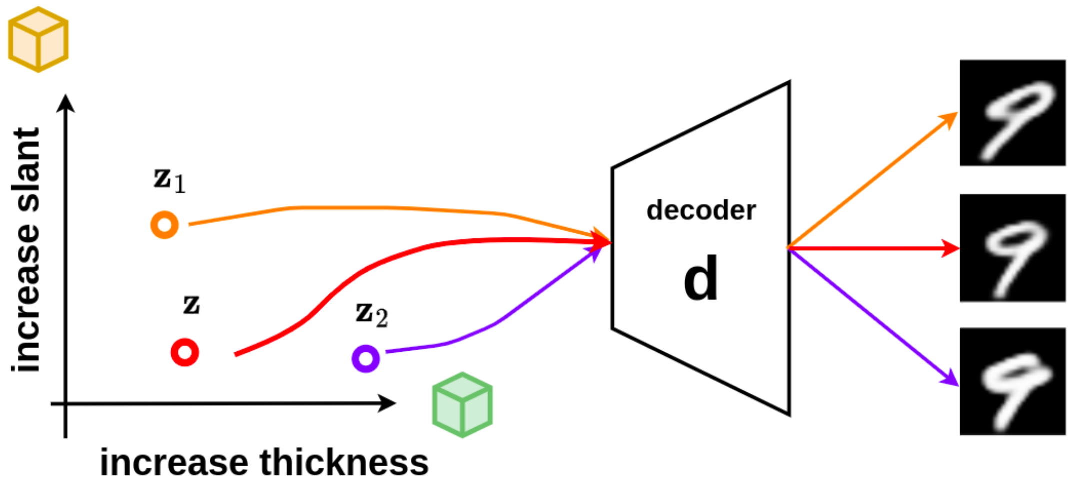
- **AR-VAE: attribute-based regularization of VAE latent space**
  [Pati, Neural Comp. Appli., 2021]

  - **Generation of structured latent space**
    - → **Specific continuous-valued attributes forced to be encoded along specific dimensions**
    - → $Loss = VAE\ loss + Attribute\ Regularisation\ Loss$



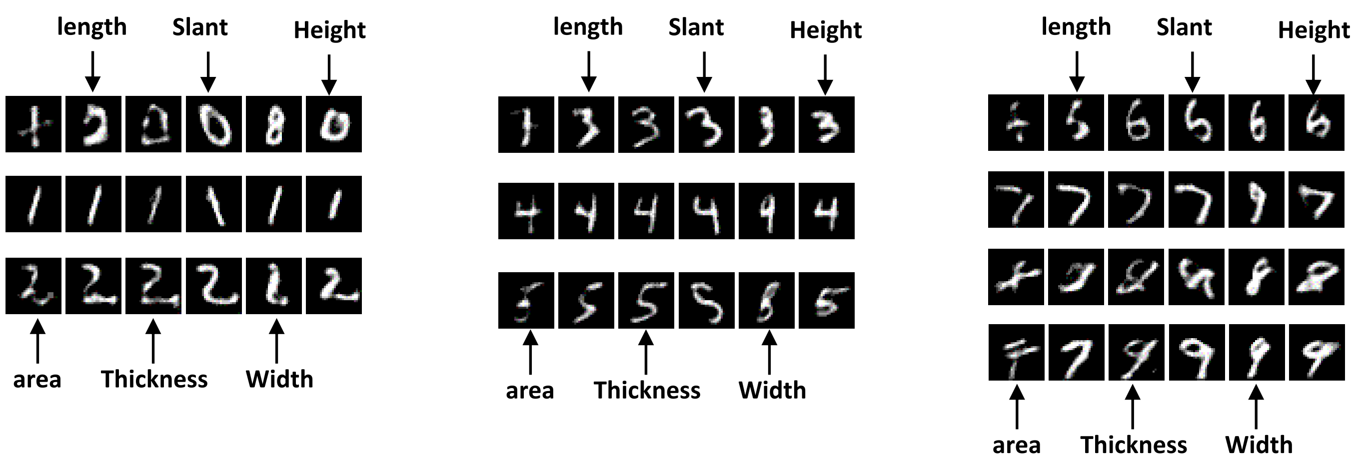
- **AR-VAE: attribute-based regularization of VAE latent space**
  [Pati, Neural Comp. Appli., 2021]

  - **Sampling of the structured latent space**



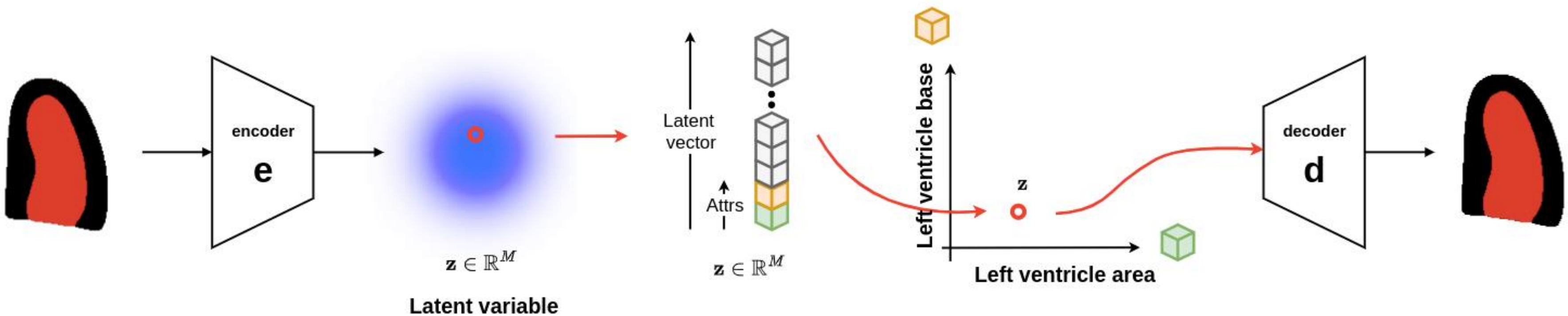
AR-VAE: attribute-based regularization of VAE latent space
[Pati, Neural Comp. Appli., 2021]

- **Sampling of the structured latent space**
  - ➔ Specific attribute (from left to right): area, length, thickness, slant, width, height
  - ➔ Each column corresponds to traversal along a regularized dimension

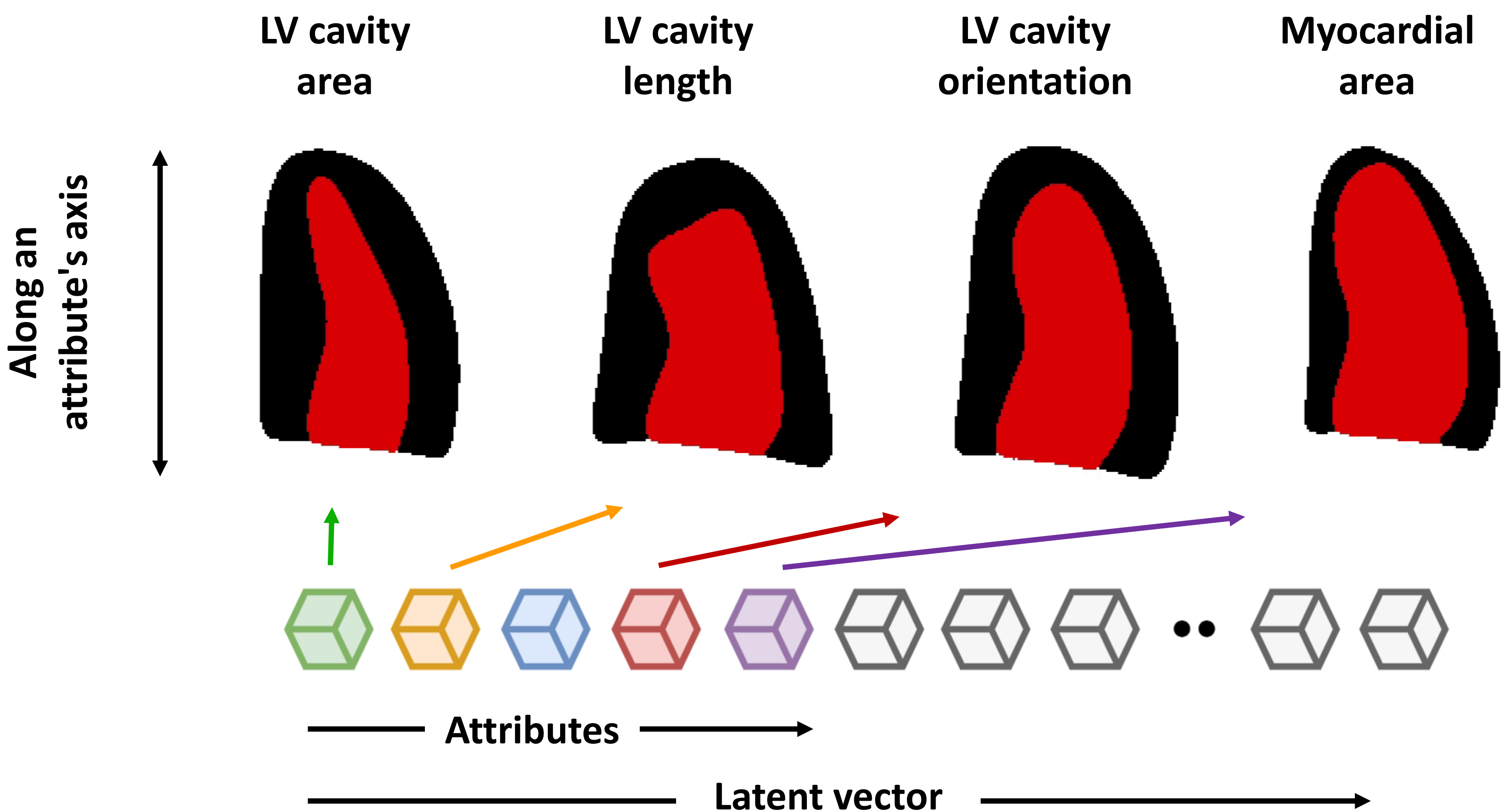


Application to the description of the cardiac shapes

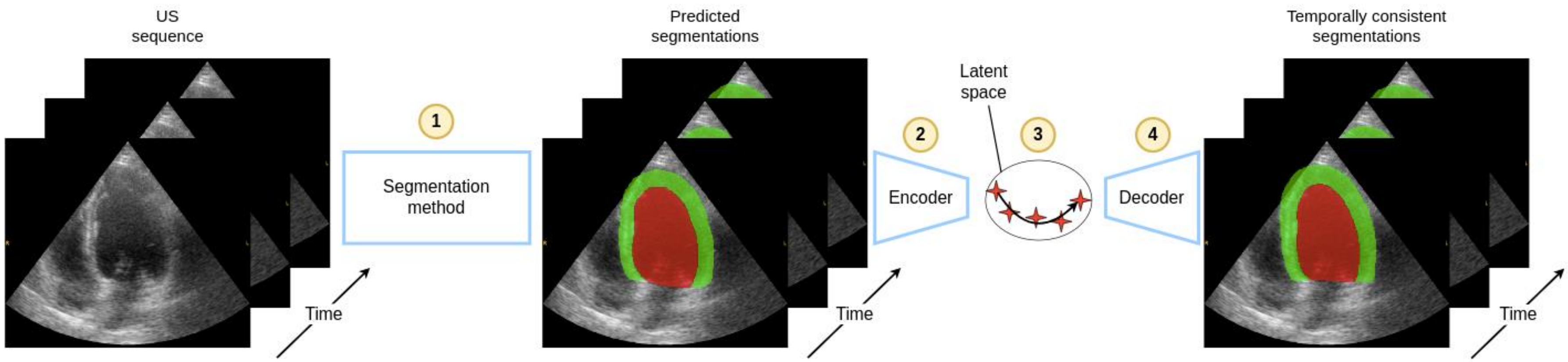
- **Generation of structured latent space according to the following attributes**
  - → Left ventricle (LV) cavity: area, length, basal width, orientation
  - → Myocardial area
  - → Epicardial center



 [Painchaud, IEEE TMI, 2022]

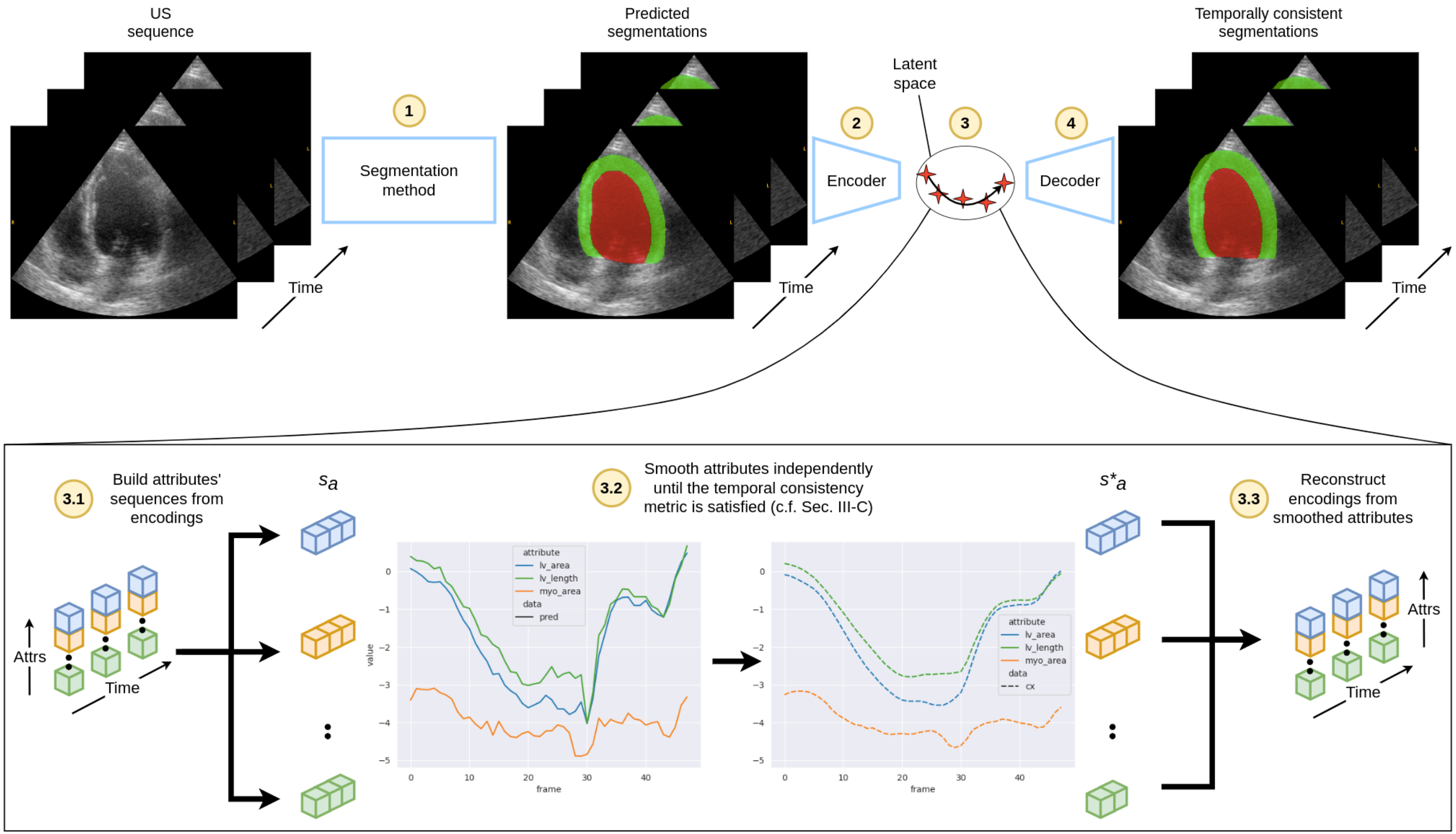


Proposed temporal pipeline



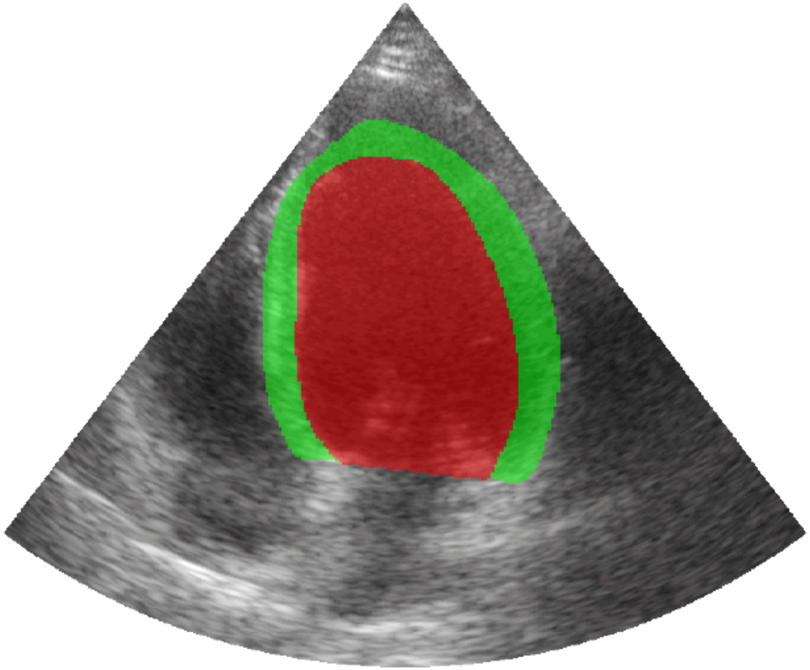
Cardiac segmentation with temporal consistency

[Painchaud, IEEE TMI, 2022]

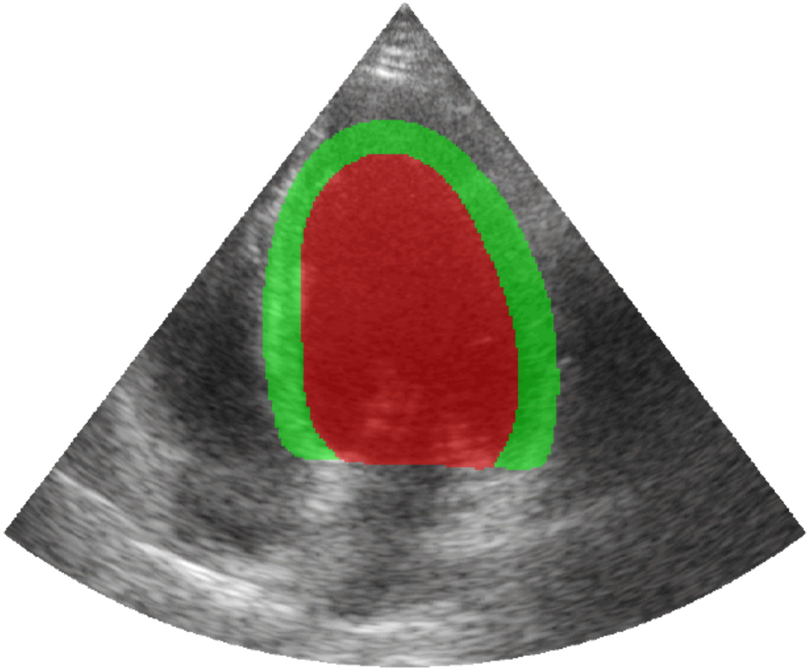


- Some post-processing examples

Original U-Net

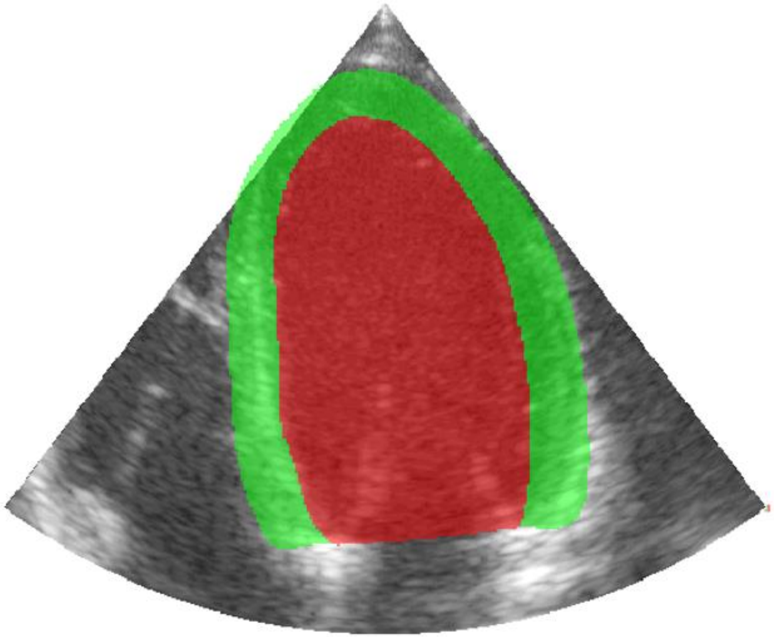


Post-processed U-Net

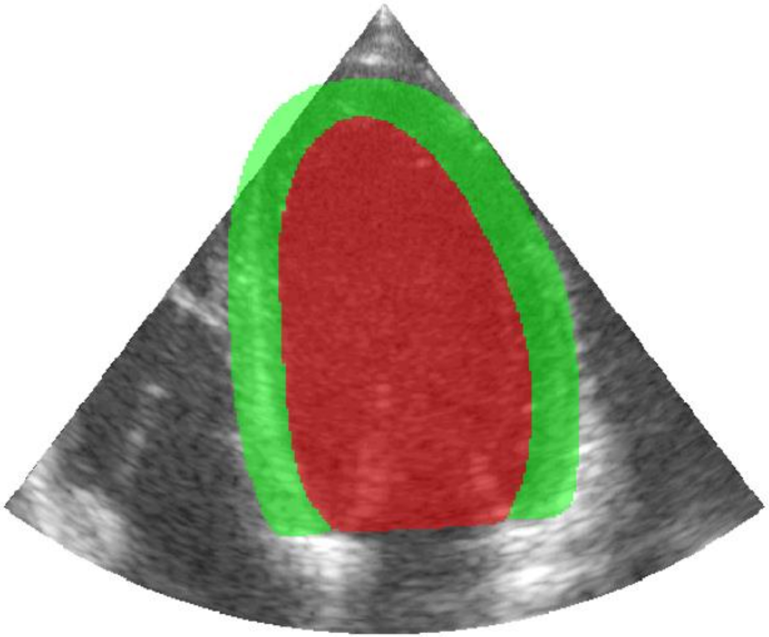


- Some post-processing examples

Original U-Net



Post-processed U-Net



To conclude

To conclude

- **VAEs can be used effectively in medical imaging**
  - **Guarantee anatomical coherence** ✓
  - **Guarantee temporal consistency** ✓
  - **Estimation uncertainty for image segmentation** ✓
  - **Generative interest limited to simple distribution**

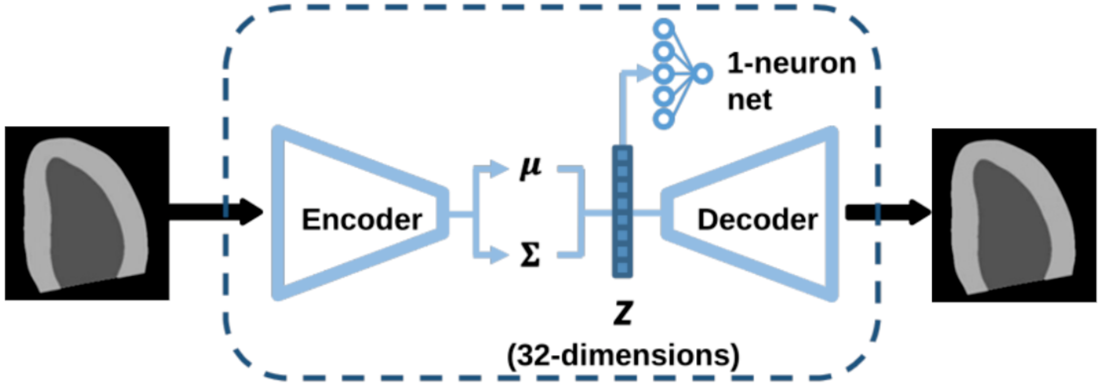
- **Useful tool for characterizing populations**
  - **Need to properly structure the learned latent space**
  - **Need to work on relatively large cohorts**

Appendix

How to guarantee the anatomical coherence ?

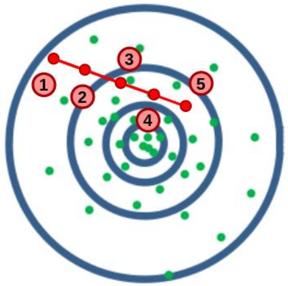
- **Constrained Variational Auto Encoder**
  - **Approximation of a latent space with local linear properties**

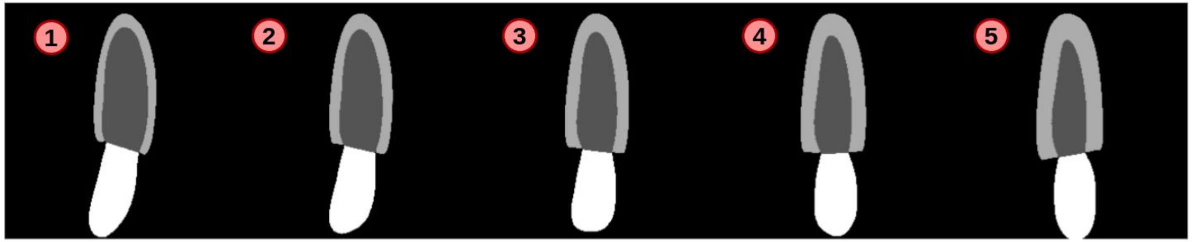
Use of a 1-neuron net to reinforce the linearity of the latent space



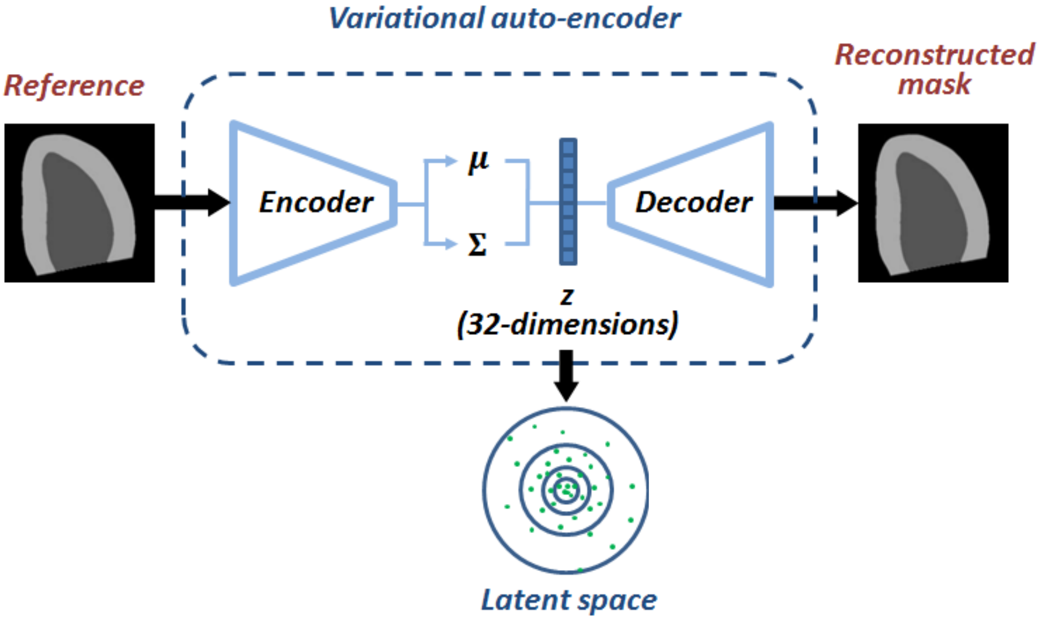
- **Constrained Variational Auto Encoder**
  - **Approximation of a latent space with local linear properties**
    - → **Linear interpolation in the latent space makes sense**

Constrained latent space



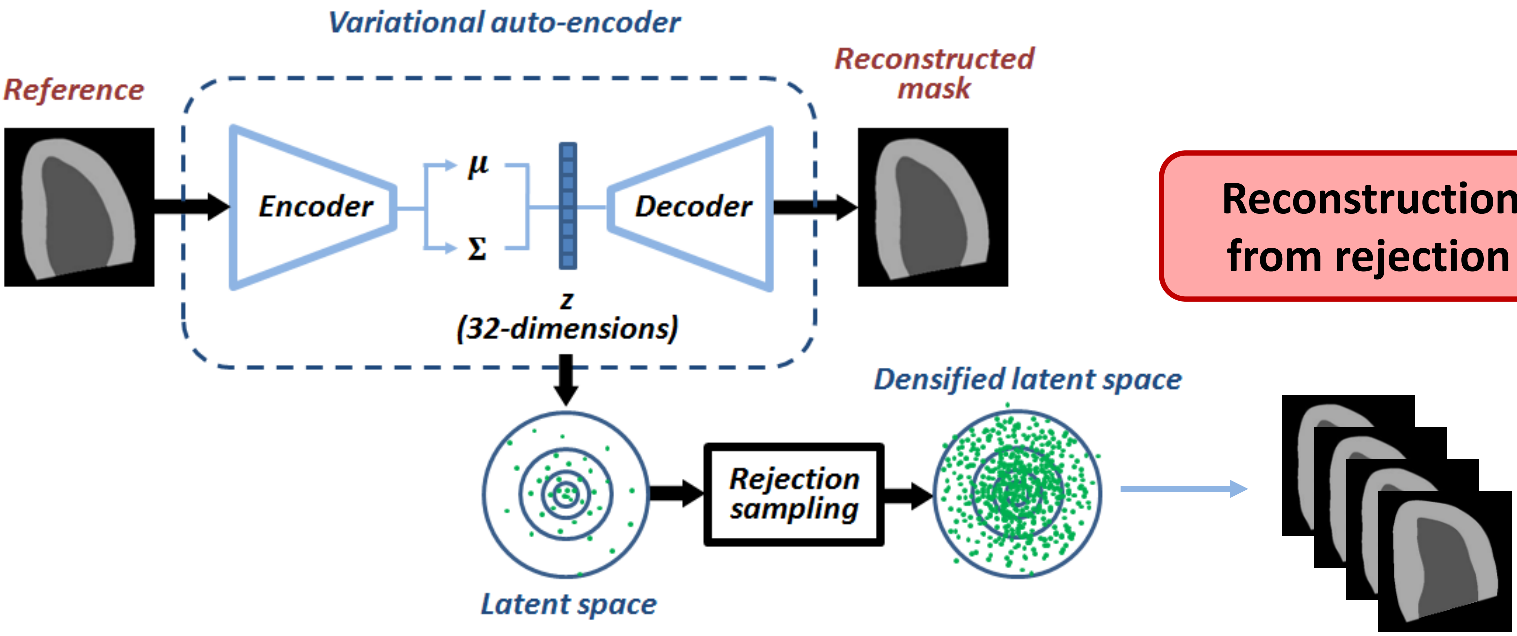


Cardiac segmentation with strong anatomical guarantees *[Painchaud, IEEE TMI, 2020]*



Efficient encoding of anatomical shapes in a latent space

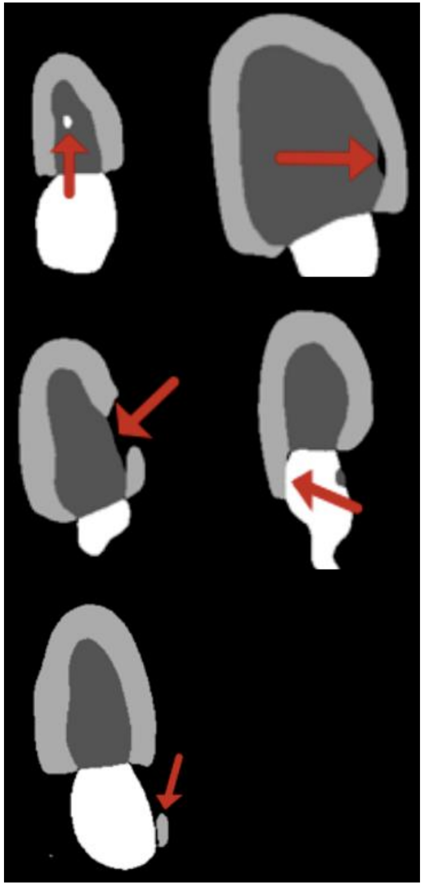
Cardiac segmentation with strong anatomical guarantees *[Painchaud, IEEE TMI, 2020]*



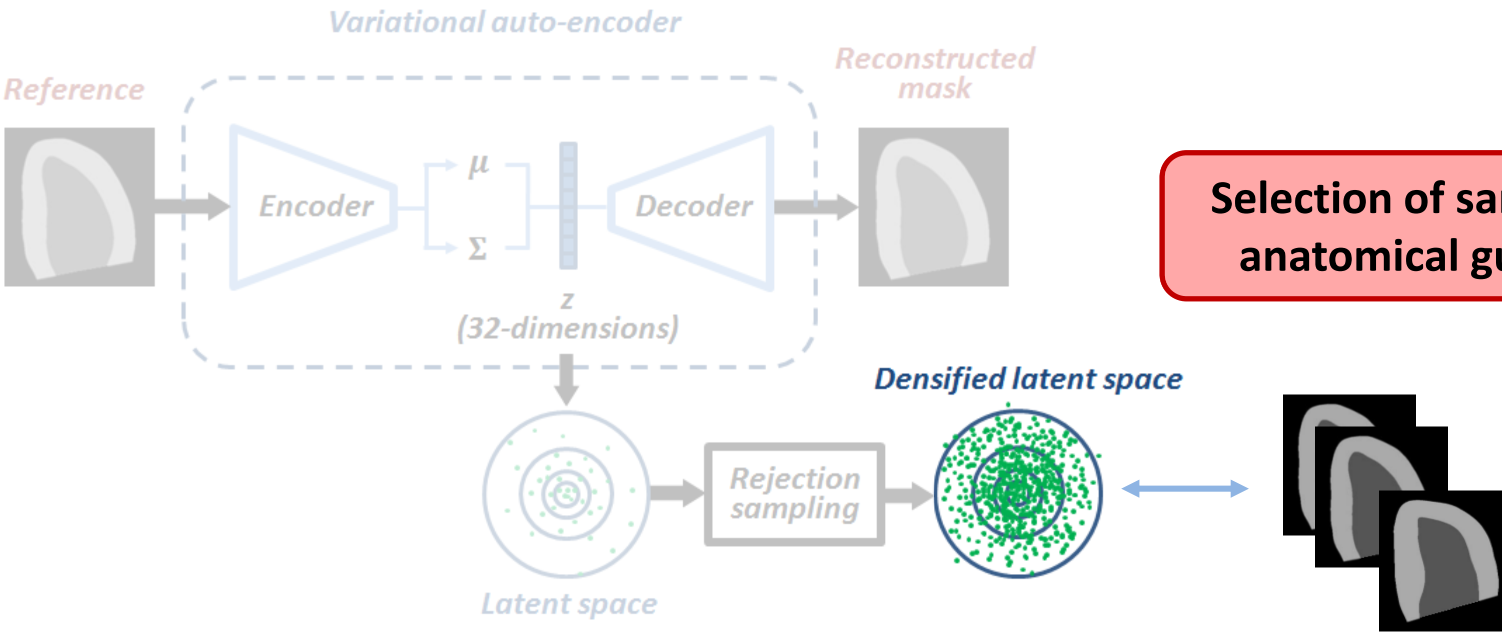
Reconstruction of masks from rejection sampling

Densified latent space with 5 million points

- **Definition of 12 anatomical metrics**
  - **(3 criteria) hole(s) in the LV, RV or LA**
  - **(2 criteria) hole(s) between LV and MYO or between LV and LA**
  - **(3 criteria) presence of more than one LV, MYO or LA**
  - **(2 criteria) size of the area by which the LV touches the background or the MYO touches the LA**
  - **(1 criterion) ratio between the minimal and maximal thickness of the MYO**
  - **(1 criterion) ratio between the width of the LV and the average thickness of the MYO**



Cardiac segmentation with strong anatomical guarantees *[Painchaud, IEEE TMI, 2020]*

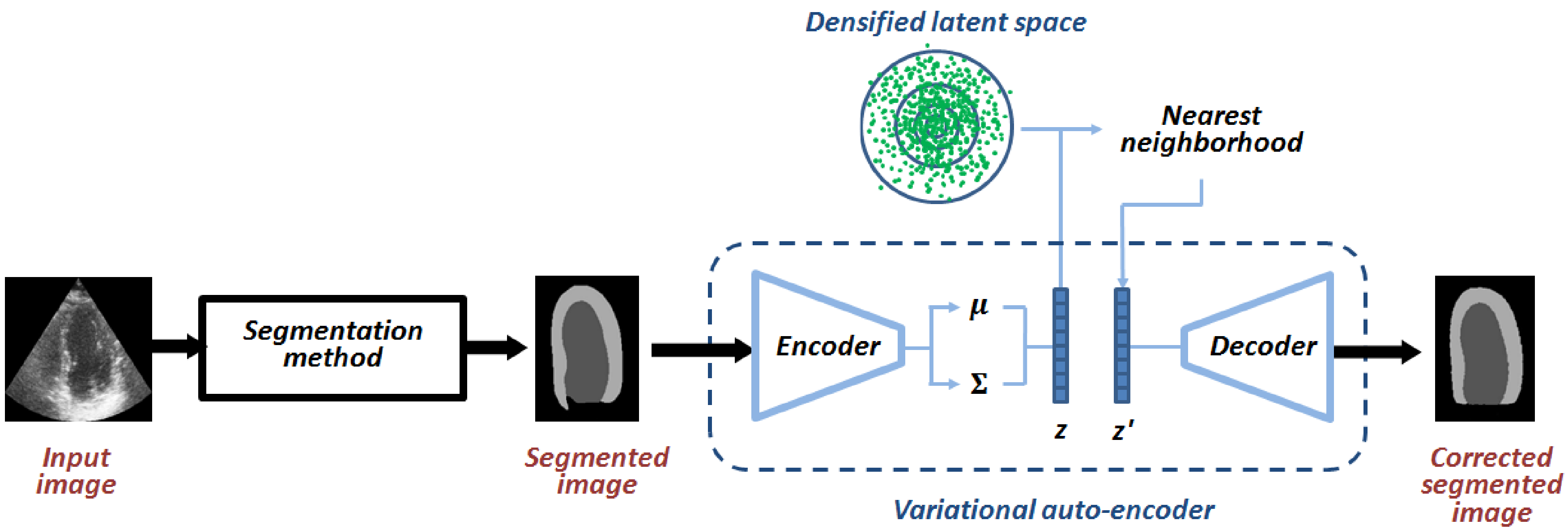


Selection of samples with anatomical guarantees

Densified latent space with 4 million points

Cardiac segmentation with strong anatomical guarantees [Painchaud, IEEE TMI, 2020]

Correction of segmentation to guarantee the plausibility of anatomical shapes → Almost same accuracy as the original methods but with correct anatomical shapes

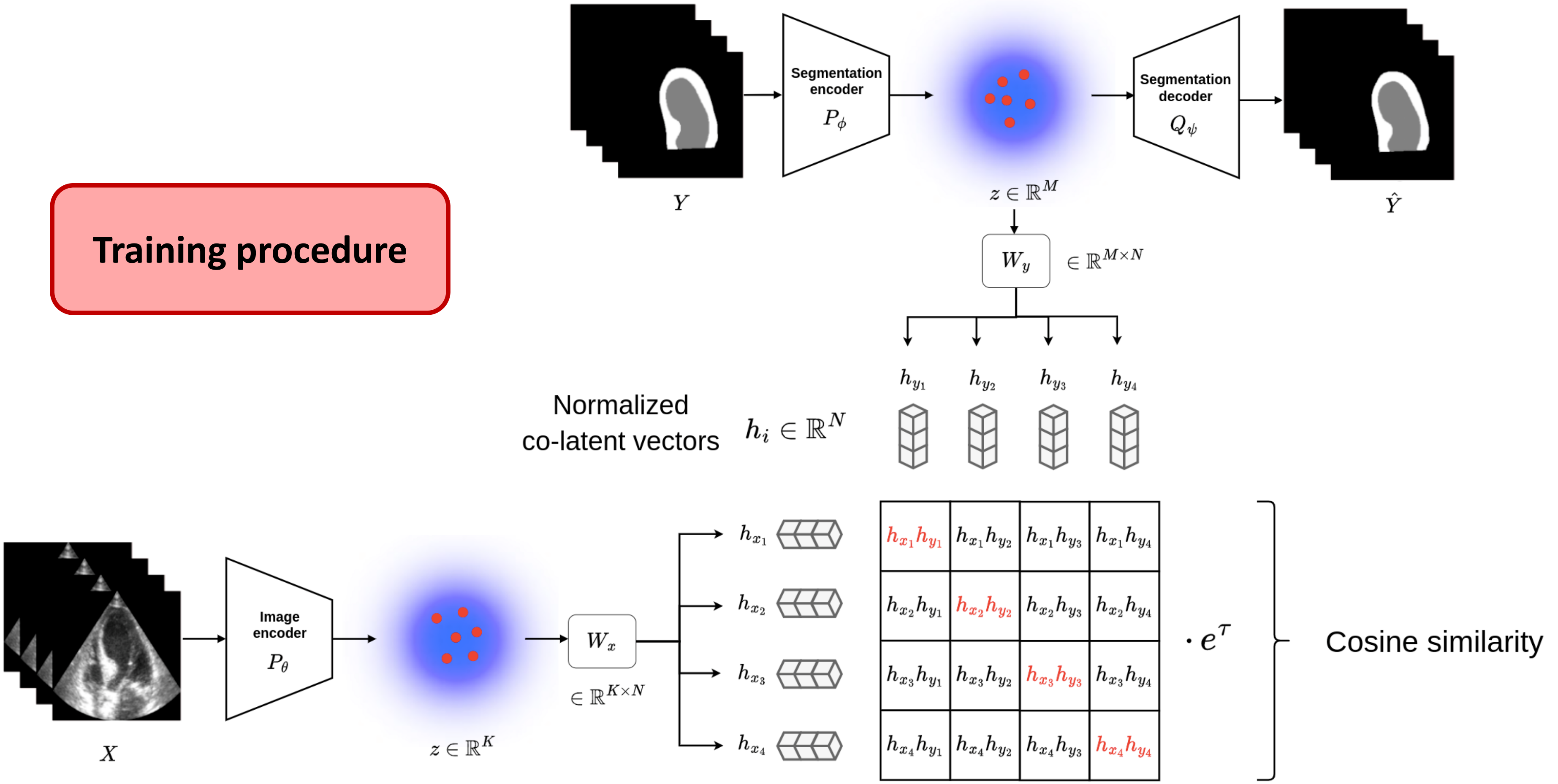


Uncertainty estimation for cardiac image segmentation

Uncertainty estimation for image segmentation

[MICCAI, 2022]

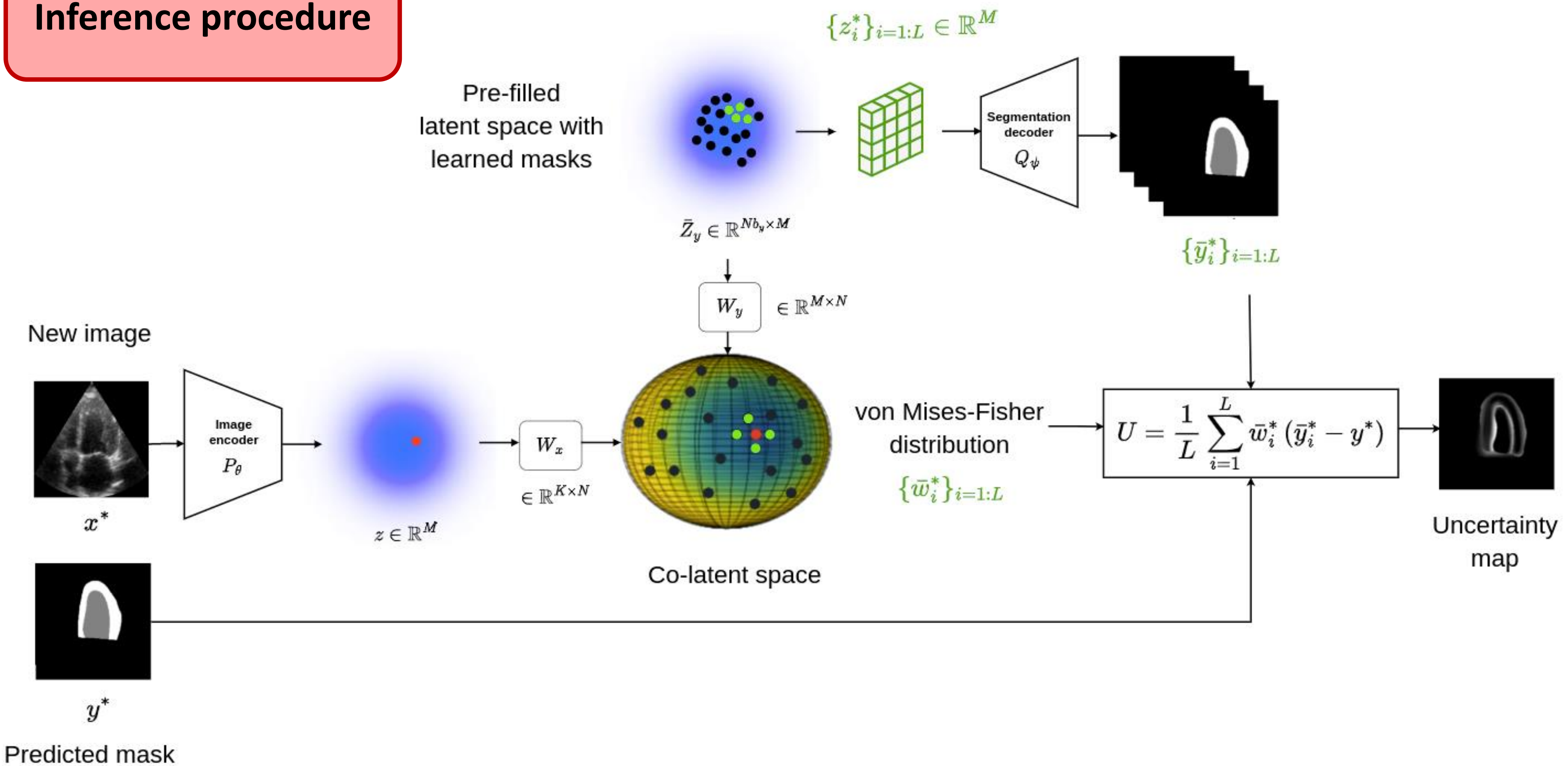
Training procedure



Uncertainty estimation for image segmentation

[MICCAI, 2022]

Inference procedure



- **Uncertainty results**